Assignment 5

Q7. $\begin{cases} \mu = 24 \\ \sigma^2 = 144 \end{cases} \Rightarrow \begin{cases} \alpha\beta = 24 \\ \alpha\beta^2 = 144 \end{cases} \Rightarrow \begin{cases} \alpha = 4 \\ \beta = 6 \end{cases}$

$X \sim \text{Gamma}(4, 6)$.

(a) $P(12 \le X \le 24)$

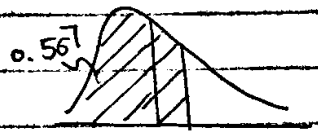
$= F(24; 4, 6) - F(12; 4, 6)$

$= F(4; 4) - F(2; 4)$

$P(X \le x) = F(x; \alpha, \beta) = F(\frac{x}{\beta}; \alpha)$

∴ Look at table A4

(b) $P(X \le 24) = F(4; 4) = 0.56$.



∴ Median is less than Mean 24.

(c) $F(\frac{x}{\beta}; \alpha) = F(\frac{x}{6}; 4) = 0.99$.

(d). We want a value t for $P(X > t) = 0.005$

$\Rightarrow P(X \le t) = 1 - P(X > t) = 0.995$

$\therefore F(\frac{t}{6}; 4) = 0.995$

$\Rightarrow$ In table A4, $F(11; 4) = 0.995$

$\Rightarrow t = 11(6) = 66$

Q3 Using Normal approximation to binomial distn.

$X \sim Bin(1000, 0.03)$

$Y \sim N(np, np(1-p)) = N(30, 5.394^2)$

(a) Using Continuity Correction,

$$P(X \geq 37) = 1 - P(X < 37)$$

$$= 1 - P(X \leq 36)$$

$$= 1 - P(Y \leq 36.5)$$

$$= 1 - P\left(Z \leq \frac{36.5 - 30}{5.394}\right)$$

(b) 6% of 1000 is 60,

$$P(X \leq 60) = P\left(Z \leq \frac{60.5 - 30}{5.394}\right)$$

Q4. $X \sim$ cycle time in min

$X \sim N(44, 20^2)$.

(a) $P(X>60 \mid X>50) = \dfrac{P(X>60,\ X>50)}{P(X>50)}$

$= \dfrac{P(X>60)}{P(X>50)}$

$P(X>60) = 1 - P(Z \le \frac{60-44}{20})$ $\quad P(X>50) = 1 - P(Z \le \frac{50-44}{20})$

$= 1 - \Phi(0.8)$ $\quad = 1 - \Phi(0.3)$

$= 1 - 0.788$ $\quad = 1 - 0.618$

$= 0.212$. $\quad = 0.382$.

$\Rightarrow P(X>60 \mid X>50) = \dfrac{P(X>60)}{P(X>50)} = \dfrac{0.212}{0.382} = 0.5545$

Q5. $X \sim$ # of students drop out.

$X \sim$ Binomial $(n=1828,\ p=0.29)$.

Using Normal approximation to calculate

$Y \sim N(np,\ np(1-p)) = N(530.12,\ 19.4^2)$.

$P(X>617) = 1 - P(X \le 617)$

$= 1 - P(Y \le 617.5)$ (Continuity correction.)

$= 1 - P(Z \le \dfrac{617.5 - 530.12}{19.4})$

$= 1 - \Phi(4.5)$

$= 0$.